Neutrosophic Modeling and Control

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Abstract--- **Quite recently, Neutrosophic Logic has been proposed by Florentine Smarandache which is based on non-standard analysis that was given by Abraham Robinson in 1960s. Neutrosophic Logic was developed to represent mathematical model of uncertainty, vagueness, ambiguity, imprecision, incompleteness, inconsistency, redundancy and contradiction in data [4],[5],[6],[7]. All the factors stated are very integral to human thinking, as it is very rare that we tend to conclude/judge in definite environments. This paper discusses how neutrosophic logic can be utilized for modeling and control for which block diagram of neutrosophic inference system is proposed, to illustrate this designing of relatively simple neutrosophic classifier has been attempted. Current problems and future directions for neutrosophic approaches are also addressed.**

Keywords--- neutrosophic logic, neutrosophic control, neutrosophic modeling

I. INTRODUCTION

Prof. L.Zadeh had revolutionized the field of logics by proposing a novel multi-valued logic, fuzzy logic in 1965 [14], where each element in fuzzy set has a degree of membership. It also has the provision of allowing linguistic variables whose truth values may vary between 0 and 1; in contrast to two values of classical Boolean logic [9].

L-fuzzy sets were suggested by Gougen in 1967 [11] in which any L-fuzzy set A is associated with a function μ_A from the universe X to lattice L, Interval-valued fuzzy set (IVFS) which are special case of L-fuzzy sets were given by Sambuc in 1975 [19] and further by Turksen [10], Belnap in 1977 defined four-valued logic [16] to cope with multiple information sources, with parameters truth (T), false (F) , unknown (U), and contradiction (C).

Fuzzy logic was extended by K. Atanassov in 1983 [12] to Intutionistic fuzzy sets on universe X where for any set A, $x \in X$, $\mu_A(x) + \nu_A(x) \leq 1$, where $\mu_A(x)$, $\nu_A(x)$ refers to membership and non-membership values, respectively of element x to set A.

Vague sets defined by Gau and Buehrer in 1993 [20], are characterized by a truth and false membership functions and is a set of objects, each of which has a grade of membership whose value is a continuous subinterval of [0,1]. Concept of interval-valued intuionistic fuzzy set (IVIFS) was given by Atanassov in 1999[12], where in IVIFS on universe X an object A is defined such that

$A=\{(x, M_A(X), N_A(x)), x \in X\}$,
with $M_A : X \rightarrow \text{Int}([0,1])$ and $N_A : X \rightarrow \text{Int}([0,1])$
and for all $x \in X$, $\sup M_A(X) + \sup N_A(x) \leq 1$

Fuzzy logic is a dominant entry in the domain of logics, which has been time and again tested and proved by many researchers that it holds the potential of generating exact results from imprecise data. Computer simulations of real world working and human interpretations is an active thrust area; where in computers are handicapped to manipulate only precise valuations. This paper is written with the aim of suggesting neutrosophic logic that has the potential of replacing all sorts of logics as it is a generalized logic which encompasses all other logics as its special instances. Neutrosophic logic is a better option to simulate human brain working which is equipped with dealing with uncertainties and vagueness.

II. DEFINITION OF NEUTRSOPHIC SET

A logic in which each proposition is estimated to have the percentage of truth in a subset *T,* the percentage of indeterminacy in a subset *I*, and the percentage of falsity in a subset *F*, is called neutrosophic logic [4].

Let *T, I, F* be standard or non-standard real subsets of $]^-0, 1^+[$,

with
$\sup T = t\_sup$, $\inf T = t\_inf$,
$\sup I = i\_sup$, $\inf I = i\_inf$,
$\sup F = f\_sup$, $\inf F = f\_inf$, and

$n\_sup = t\_sup+i\_sup+f\_sup$,
$n\_inf = t\_inf+i\_inf+f\_inf$. and

$n\_inf = \inf T + \inf I + \inf F \geq {}^-0$, and
$n\_sup = \sup T + \sup I + \sup F \leq 3^+$.

The sets *T, I, F* are not necessarily intervals, but they may be any real sub-unitary subsets; discrete or continuous; single-element, finite, or (countable or uncountable) infinite; union or intersection of various subsets etc [4], [5], [6],[7].They may also overlap. We use a subset of truth (or indeterminacy, or falsity), instead of a number only, because in many cases we are not able to exactly determine the percentages of truth and of falsity but approximate them: for example a proposition is true between 30-40% and between 60-70% false, even worst: between 30-40% or 45-50% true (according to various analyzers), and 60% or between 66-70% false [4].

Other advantage of utilizing Neutrosophic logic is that in comparison to other logics, it is not confined to the range of [0,1], and well distinguishes between absolute true/false values from relative true/false values. Neutrosophic logic has the provision of assigning >1 as well as <1 values to its neutrosophic components, *(t, i, f)*. So whenever for any tautology values of *t/i/f* >1, this would imply absolute true/indeterminate/false similarly whenever values of *t/i/f* <1, this would imply conditional(relative) truth/indeterminacy/falsity. This mechanism of assigning over boiling values (>1) or under dried values (<0) helps in justifying dissimilarity between unconditionally true ($t > 1$, and $f < 0$ or $i < 0$) and conditionally true propositions ($t \leq 1$, and $f \leq 1$ or $i \leq 1$) [6],[7].

III. NEUTROSOPHY'S ANALYTICAL COMPARISON TO OTHER LOGICS

Neutrosophic logic is far better representation of real world data/executions because of the following reasons:

a. Fuzzy logic though ensures multiple belongingness of a particular element to multiple classes with varied degree but capturing of neutralities due to indeterminacy is missing, it is further limited by the fact that membership and non-membership value of an element to a particular class should sum up to 1[2],[13],[14].

b. Similarly other allied logics like Lukasiewicz logic considered three values (1, 1/2, 0), Post considered m values, etc, but all are handicapped with the constraint that values can vary in between 0 and 1 only[15].

c. Intuitionistic fuzzy logic though deals with indeterminacy parameter related to a particular element, but this fact is still constrained with the condition that , for any element x , indeterminacy value (x)=1 – [membership value(x)+ non-membership value(x)]. There is no provision of distinguishing between relative and absolute truth/indeterminacy/falsity [12].

d. In a rough set, an element x on the boundary-line cannot be classified as a member of a particular class nor of its complement with certainty[17],[18]; but can be very well described by neutrosophic logic , such that *x (T,I,F)* where *T,I,F* are standard or non-standard subsets of the nonstandard interval $]^-0, 1^+[$

When the sets are reduced to an element only respectively, then

$t\_sup = t\_inf = t$, $i\_sup = i\_inf = i$, $f\_sup = f\_inf = f$,
and $n\_sup = n\_inf = n = t+i+f$

Hence, the neutrosophic logic generalizes:

- the *Boolean logic* (for $n = 1$ and $i = 0$, with t, f either 0 or 1)
- the *multi-valued logic* (for $0 \leq t, i, f \leq 1$)
- the *fuzzy logic* (for $n = 1$ and $i = 0$, and $0 \leq t, i, f \leq 1$)
- the *Intuitionistic fuzzy set*, for ($t+i+f$=1 and $0 \leq i < 1$)

IV. NEUTROSOPHIC CONTROLLERS

Proposed neutrosophic controller would be a special system which would be more generalized and indeterminacy tolerant in it's working as compared to fuzzy counterparts. Neutrosophic systems controllers as proposed would vary substantially according to the nature of the control problems that they are supposed to solve. Here we restrict ourselves to the explanation of relatively simple classifier problems.

Neutrosophic systems similar to their fuzzy counterparts would be capable of utilizing knowledge obtained from human operators. In majority of the real world controllers it is difficult to devise a precise mathematical model that would simulate system behavior; also it is unlikely that the data acquired by the system would be 100% complete and determinate. Incompleteness and indeterminacy in the data can arise from inherent non-linearity, time-varying nature of the process to be controlled, large unpredictable environmental disturbances, degrading sensors or other difficulties in obtaining precise and reliable measurements. Humans can take intelligent decisions in such situations. Though this knowledge is also difficult to express in precise terms, an imprecise linguistic description of the manner of control can usually be articulated by the operator with relative ease.

This important concept of range of neutralities is missing in fuzzy logic controller and other allied logics, as fuzzy logic controller is concerned about membership and non membership of a particular element to a particular class [1];

and does not deals with indeterminate nature of data acquired that could happen due to various reasons discussed above; also the concept of fuzzy logic is paralyzed with the fact that non-membership value=1-membership value. So to deal with such situations wherein there is possibility of indeterminacy and incompleteness in the data acquired, neutrosophic controller is proposed.

It is suggested that neutrosophic controller be constituted of 4 modules.

a. Neutrosophication module
b. Neutrosophic rule base
c. Neutrosophic engine
d. De-neutrosophication module

Arrangement of these modules have been represented as in fig. 1.

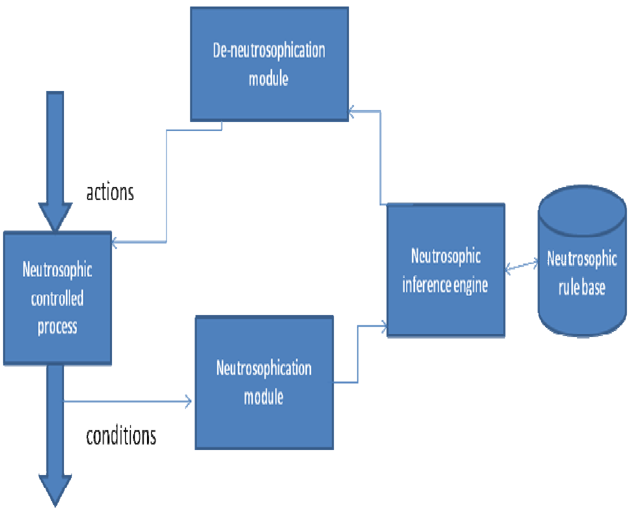

Fig. 1: Block diagram of proposed neutrosophic controller

Algorithm of the proposed neutrosophic controller is as stated below:

Step1: Record the measurements of all the variables that represent relevant conditions of the controlled process.

Step2: The acquired measurements are then converted to appropriate neutrosophic sets to capture the measurement truth, falsity and indeterminacy using truth, falsity and indeterminacy membership functions respectively. This step is called as neutrosophication step. Same has been shown in figure2.

Step3: Neutrosophied measurements are then used by the inference engine to evaluate the control rules stored in the neutrosophic rule base. This evaluation will result in a neutrosophic set or several neutrosophic sets which would be defined on the universe of possible actions.

Step 4: This neutrosophic set is then converted, in the final step of cycle, into a single (crisp) value, having triplet format like *x (t, i, f)*; which would be the best representative of the derived neutrosophic set. This process is called as de-neutrosophication.

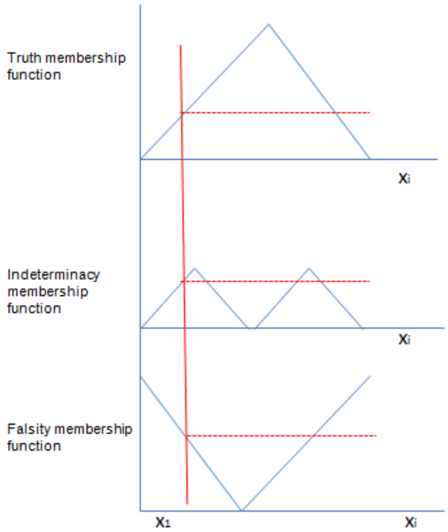

Fig. 2: Neutrosophication process

Next section is dedicated to the explanation of relatively simple classifier using MATLAB.

V. EVALUATION DATASET

This paper utilizes iris dataset (http://archive.ics.uci.edu/ml/datasets/Iris). All experiments have been carried out on MATLAB 7.0. Iris dataset consists of 4 attributes; sepal length, sepal width, petal length and petal width and is having 150 instances which are categorized into 3 classes; iris-setosa, iris-versicolor and iris-virginica. 30 instances from each class have been used for training (for making rule set) and 20 from each class have been used for testing.

VI. E XAMPLE :WHAT WE SUGGEST?

As mentioned above that this paper utilizes iris dataset which is simulated using MATLAB 7.0. Currently MATLAB doesn't provide with the facility of neutrosophication, so in an attempt to simulate it, three FIS have been designed with the name of iris-t, iris-i, iris-f, representing true, indeterminate and false value, which can be executed independent of each other [8].

This paper represents the working of mamdani type NIS: neutrosophic inference system. Table 1, discusses the specifications of neutrosophic classifier build using Fuzzy toolbox of MATLAB 7.0.

TABLE 1: NIS Specifications

Neutrosophic Inference system (iris-t, iris-i, iris-f)	Mamdani
And method	Product
Implication method	Product
Aggregation method	Sum
Deneutrosophication	Centroid
Membership functions	Triangular, Trapezoidal [0,1]

This paper targets to simulate working of the proposed neutrosophic classifier. It has been suggested on the lines of neutrosophic logic such that the output value should take the neutrosophic format of the type: Output(true, indeterminacy, false),which is contrary to one de-fuzzified output value generated by fuzzy inference system.
Figure 3, 4,5 and 6, 7,8 represents the iris-true FIS (iris-t) and iris-indeterminate (iris-i) FIS respectively. They will generate true and indeterminate components respectively of neutrosophic triplet. On the same lines iris-false (iris-f) is also designed, that gives third component of neutrosophic triplet.

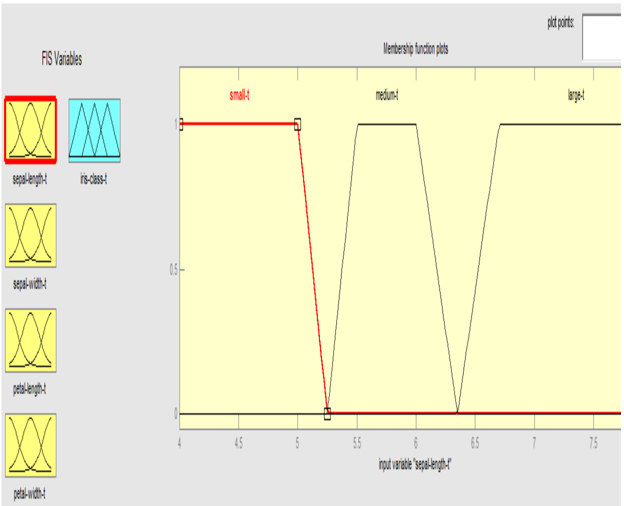

Fig. 3: True membership function for sepal length, represented as sepal-length-t

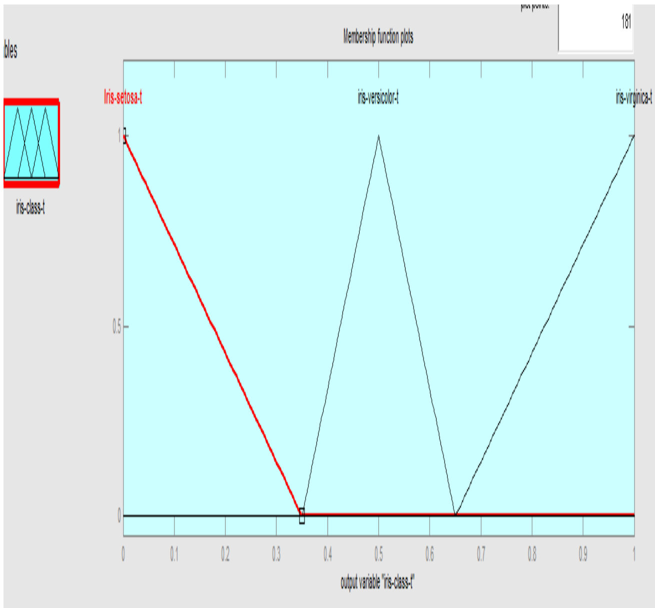

Fig. 4: True membership function for iris classes, represented as iris-class-t

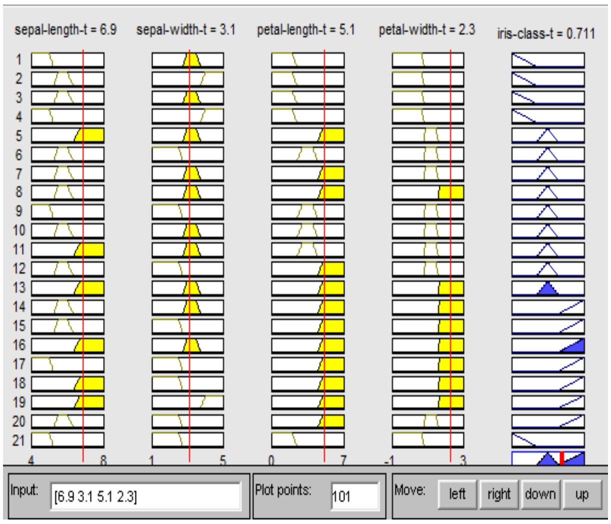

Fig. 5: Rule viewer details of iris-t for 142[nd]instance (6.9, 3.1,5.1, 2.3) giving de-neutrosophied value of 0.711

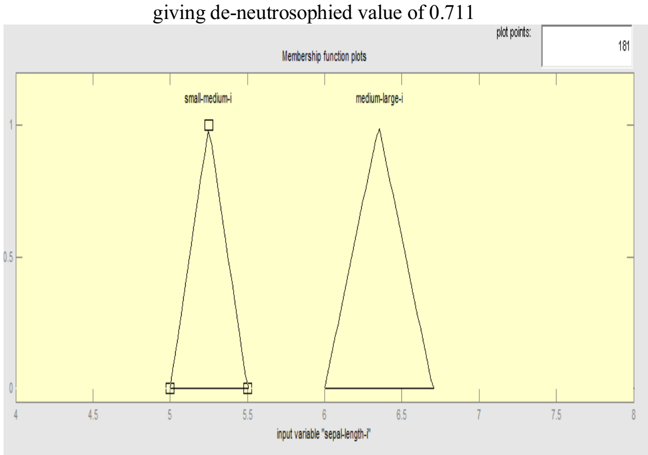

Fig.6: Indeterminate membership function for sepal length, represented as sepal-length-i

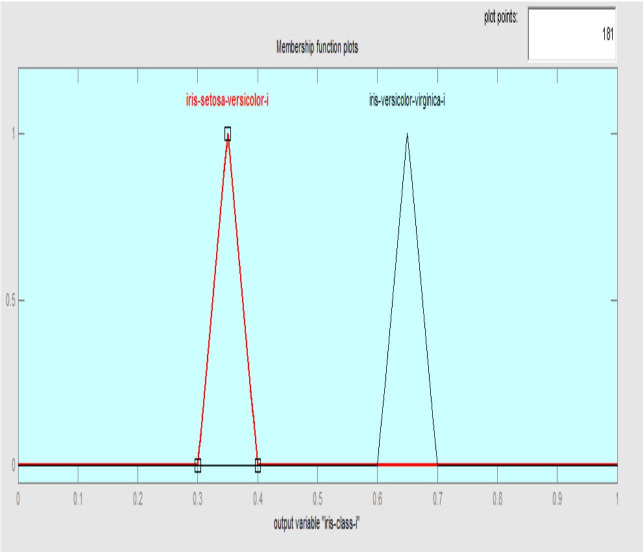

Fig.7: Indeterminate membership function for iris classes, represented as iris-class-i

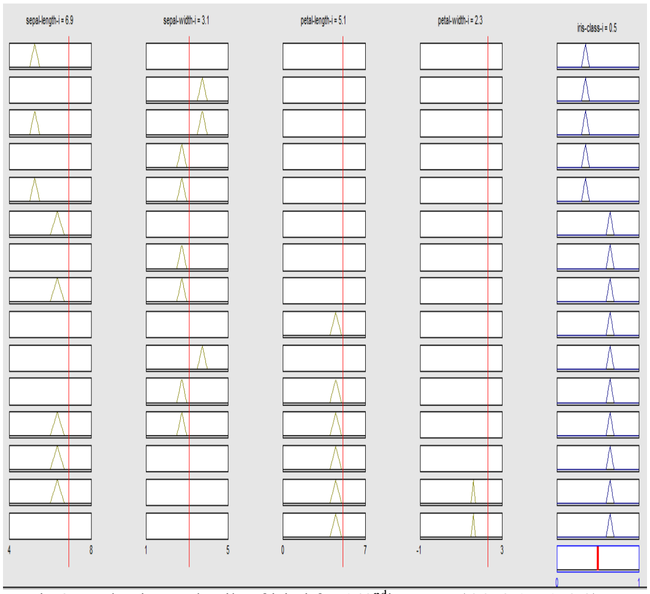

Fig.8: Rule viewer details of iris-i for 142ndinstance (6.9, 3.1,5.1, 2.3) giving de-neutrosophied value of 0.5

i. De-neutrosophied truth value of 142nd entry for iris-t is 0.711 which lies clearly in iris-virginica, as shown in figure 5.

ii. For iris-i, that represents indeterminate component of neutrosophication process, membership functions have been designed only for the ranges where there was overlapping, say for example in sepal-length; overlapping was from 5-5.5, so small-medium-i membership function is designed that captures the indeterminacy spanned in this region for sepal-length-i, as shown in figure 6. Same has been done for iris class membership function in iris-i, as shown in figure 7.

iii. De-neutrosophied indeterminacy value of 142nd entry for iris-i is 0.5 in figure 8 clearly indicates that for this entry there is no indeterminacy associated with it. Following indeterminacies were recorded for the following instances:

   a. 32,33,40,99 gave indeterminacy of setosa-versicolor-i=0.35

   b. 87-90,92,96,137-139,147-149 gave indeterminacy of versicolor-virginica-i=0.65

iv. Same results were recorded for iris-f (false component of neutrosophic component). Membership functions for iris-f were designed similar to iris-i, but with a difference that height of all the membership functions is 0.5.

So for 142nd entry de-neutrosophied value is (0.711, 0.5, 0.5) which is interpreted as (0.711 degree of membership in iris class virginica, zero indeterminacy, zero falsity), as 0.5 value is not spanned by any of the designed indeterminacy or falsity functions.

By our observation, designing reasonable neutrosophic membership functions and choosing reasonable training data set which is based on specific application domain can reduce the prediction error a lot. Here the example used is just for illustration.

VII. CONCLUDING REMARK

A. Current problems and possible solutions

Artificial intelligence domains like expert systems, decision support system, medical reasoning, belief system, information fusion tends to rely not only on the evidence supported by truth value, but also on falsity membership against evidence. IFS and interval IFS were successful in capturing the falsity of the belief but were limited in their applications due to the restriction that $t^+ + f^+ \leq 1$,this indicates that intuitionistic fuzzy sets and interval valued intuitionistic fuzzy sets are suited to capture the incompleteness of the problem but cannot handle indeterminate information and inconsistent information which exists commonly in belief systems.

For example if an opinion of an expert is asked about certain statement, then he or she may say that the

possibility that the statement is true is between 0.5 and 0.7 and the statement is false is between 0.2 and 0.4 and the degree that he or she is not sure is between 0.1 and 0.3, this can be appropriately handled by neutrosophic logic which have the truth, indeterminate and falsity membership functions independent of each other and each can overuse or underuse the limit of [0,1], depending on the depiction of absolute or conditional values respectively.

As it is clear that all theories that proposes logical interpretations cannot ever be freed of paradoxes [3], it becomes utter mandatory and clear that a generalized logic is required that apart from representing two extremes (true and false) is capable of encompassing continuous spectrum of neutralities, that can happen due to various levels of overlapping imprecision and misunderstandings; which as of date is best represented by Neutrosophic logic.

Thus Neutrosophic set emerges as a dominant formal framework which encompasses various other logics like classic set, fuzzy set, interval valued fuzzy set, intuitionistic fuzzy set, interval valued intuitionistic fuzzy set.

B. Future Directions

As discussed in the paper that neutrosophic logic is a framework to measure the truth, indeterminacy, and falsehood, which closely resembles human psychological behavior. So current systems which are dedicated to do human brain simulations using other existent logics are constrained because of their strict conditions, as predominant ones have been discussed in section III. Due to the reasons explained; definitely neutrosophic logic holds its chance to be experimented and to be utilized for real world executions and human psychology simulations.

By employing neutrosophic logic framework we have proposed fuzzy models tailored to suit the constraints of neutrosophic logic. In this paper simple neutrosophic classifier has been designed for data classification. Due to its inherit generalized nature and flexibility of over or under stretching the limit of [0 1] , neutrosophic logic will find application in varied domains where information is vague, doubtful, partial and conflicting such as databases, web intelligence image processing, decision making , bioinformatics, expert systems and medical informatics.

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